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A complete software package for transformer design optimization and economic evaluation analysis

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Abstract. In the present paper, a Transformer Design Optimization (TDO) software package is developed providing a user-friendly transformer design and visualization environment. This software consists of a collection of design optimization, visualization and verification tools, able to provide transformer designers all the proper interactive capabilities required for the enhancement of the automated design process of a manufacturing industry.

Introduction

With the advent of digital technology, the considerable cost reduction of computer hardware has allowed software engineers the opportunity to be provided with automated support of the development process. Today's software engineering environments are integrated around software management systems that offer support for all software process activities. The decrease of delivery time is of primary importance for transformer market and can be achieved through reduction of the industrial cycle, i.e. the study-design-production time. For this purpose, suitable software systems employing appropriate tools for the automation of each phase of the industrial cycle are required, especially in cases of customer orders of small quantities and different transformer specifications [1]. In order to compete successfully in a global economy, transformer manufacturers need design software capable of producing manufacturable and optimal designs in a very short time. The first transformer design was made on computer in 1955 [2]. Several design procedures for low-frequency and high-frequency transformers have appeared in the literature after the 70's. Judd and Kressler [3] presented a technique for designing transformers with given size and type of structure to have maximum volt-ampere (VA) output while at the same time insuring the satisfaction of a number of design constraints. Poloujadoff et al. [4] show the variation in the price of the transformer depending on the primary turns, which is an approximately hyperbolic function. Jeweel [5] does a functional proposal with students in electrical engineering, in which the student designs, builds and tests a 10 VA transformer. Grady et al. [6] deal with the teaching of design of dry type transformers, based on a computer program, where the user optimizes its design based on trial and error. Furthermore, Rubaai [7] describes a computer program yielding an optimal design of a distribution transformer based on user input data. Andersen [8] presented an optimizing routine, Monica, based on Monte Carlo simulation. Basically, his routine uses random numbers to generate feasible designs from which the lowest cost design is chosen. Hernandez and Arjona [9] develop an object-oriented knowledge-based distribution transformer design system, in conjunction with FEM, which is used as a tool for design performance validation.

Deterministic methods provide robust solutions to the transformer design optimization problem. In this context, the deterministic method of geometric programming has been proposed in [10] in order to deal with the design optimization problem of both low frequency and high frequency transformers.

The present paper presents the development of a novel transformer design optimization software, based on mixed integer non-linear programming (MINLP) methodology and numerical field computation techniques. The software is used for the design of three phase, wound core, liquid immersed transformers. The presentation focuses on the main features of the software, namely the management of the transformer input data, the main parameters and crucial input data of the MINLP algorithm as well as the numerous capabilities provided for the visualization, verification and economic evaluation of the obtained optimum designs. Methodological details of the design optimization algorithm along with the novel approaches ensuring global transformer design optimization during the development of the software are described in [11]-[13]. The method is applied for the design of distribution transformers of several ratings and loss categories and the results are compared with a heuristic transformer design optimization method (which is already used by the transformer industry [14][15]), resulting to significant cost savings.

Structure of the proposed software package

The structure of the proposed software package is depicted in the flowchart of Fig. 1. Mixed integer non-linear programming consists the computational core of the software, resulting to the optimum values of the design variables, namely the number of secondary winding turns, the magnetic induction magnitude (B), the width of core leg (D) and the core window height (G) (Fig. 2), while the rest of the transformer design and operational characteristics are calculated by an algebraic design model, based on analytical equations. Upon user selection, the windings current density values can be added to the design vector or their value can be either prescribed or defined by the thermal short-circuit test method. The characteristics of the optimum design can be validated by the use of three dimensional (3D) finite element method (FEM) and visualized as plots, while mechanical drawings of the active part and tank can also be illustrated. Furthermore, economic analysis tools, based on the Total Owning Cost method [14] can be employed for the economic evaluation of the optimum design and provide the possibility to compare it with other sub-optimal solutions. A large database of optimum designs of various ratings is also available to the user, which can be used as guidelines in order to render the optimization procedure deskilled and easy to implement. All the aforementioned tools are included in a carefully designed graphical user interface (Fig. 3), accompanied by proper data management tools.

Data Input

Two groups of input data must be defined by the user, prior to the transformer design optimization process, concerning: i) the transformer technical characteristics and ii) the mixed nonlinear programming methodology parameters. These data are necessary for the execution of the algorithm and the derivation of the optimum transformer design. One of the major advantages of TDO software relies on the small amount of necessary input data for the transformer technical characteristics, while all the necessary parameters for the initialization and execution of the mixed integer nonlinear programming algorithm are predefined and do not require user interaction.

Transformer technical characteristics. The fourteen input parameters concerning the transformer technical characteristics are: nominal power, primary and secondary winding material (copper or aluminum), primary and secondary line-to-line voltage, primary and secondary winding connection type (delta, star or zig-zag), primary and secondary winding conductor type (providing the choice among single or double circular or rectangular wire and sheet), operating frequency, type of magnetic material, method for the determination of the windings cross-section (based on the thermal short-circuit test or the current density, as depicted in Fig. 1), guaranteed no-load and load losses (which can be defined according to CENELEC standard [16] or upon user selection).

The fourteen aforementioned parameters are sufficient for the derivation of the rest of the transformer characteristics, since the software implements calculations that define a significant number of other electrical and mechanical data (e.g. guaranteed short-circuit impedance, basic insulation level, windings insulation type, number of cooling ducts, details of the tank and its corrugated panels, etc.), while a certain number of design constants are predefined based on the experience of transformer design engineers in the manufacturing industry as well as experimental data on a large amount of produced and tested transformers. However, it must be noted that the possibility to access and modify these data is provided by the software, enabling more expert users on transformer engineering to examine more specialized designs.

The tolerances for the transformer no-load losses, load losses, total losses and short-circuit impedance (i.e. the maximum acceptable deviation between the respective designed and guaranteed values) are predefined as percentages of the guaranteed values, according to IEC 60076 standard [17]. However, the user may interfere and change these values, in case some different specification has to be examined.

Finally, representative values for the cost of the transformer eight main materials (namely, the primary and secondary winding material, magnetic material, insulating liquid, insulating paper, duct strips, tank sheet steel and corrugated panels material) are provided in the software and can be modified by the user.

Parameters of the mixed integer nonlinear programming algorithm. The mixed integer nonlinear programming algorithm seeks an optimum for the transformer design, defined by a set of integer variables linked to a set of continuous variables that minimize the objective function and meet the restrictions imposed on the transformer design problem. These restrictions are designated by the tolerances in the deviation between the designed and guaranteed values of losses and short-circuit impedance, as well as some manufacturing constraints. The objective function variables, i.e. the design variables are: the number of secondary winding turns, the magnetic induction magnitude (B), the width of core leg (D) and the core window height (G) (Fig. 2). Since the windings cross-section is a major factor affecting the overall transformer design, and it is linked to the windings current density, the possibility to insert the primary and secondary winding current density to the vector of the design variables is provided by the software, increasing the number of design variables from four to six. More details on various options for the definition of the current density and its importance on the optimization process are described in Section “Methods for the determination of the windings cross-section”.

The initial value of each of the four (or six) design variables, as well as the upper and the lower bound value for the four (or six) design variables, along with the choice of type of the design variables (which can assume not only continuous values but also integer values) are predefined in the software, according to the transformer main characteristics (e.g. nominal power, primary and secondary voltage, connection) as a result of a trial and error process carried out on a wide spectrum of various distribution transformer ratings during the development of the software.

The software also provides access to the values for the optimization algorithm termination criteria, consisting in the termination tolerance on the design vector, the objective function value or the constraint violation as well as the maximum number of objective function evaluations allowed.

Selection of the objective function. Three possible types of objective function can be used for the design optimization: i) the manufacturing cost, i.e. the cost of the transformer eight main materials, listed in Section “Data Input”, ii) the total owning cost using values for the loss coefficients provided by the user and iii) the total owning cost based on computed loss coefficients. For the total owning cost calculation, the user must also provide the cost of the rest of the materials (apart from the eight main materials), the labour cost and the sales margin in order to yield the final bid price (purchase cost) which is added to the loss cost (operating cost) in order to yield TOC. In the case of the third objective function, the definition of the loss cost factors is based on the IEEE Standard C57.120 [18] and additional data for the discount rate, the transformer lifetime, the electricity price, the hours of transformer operation per year and its average per unit load must be given by the user.

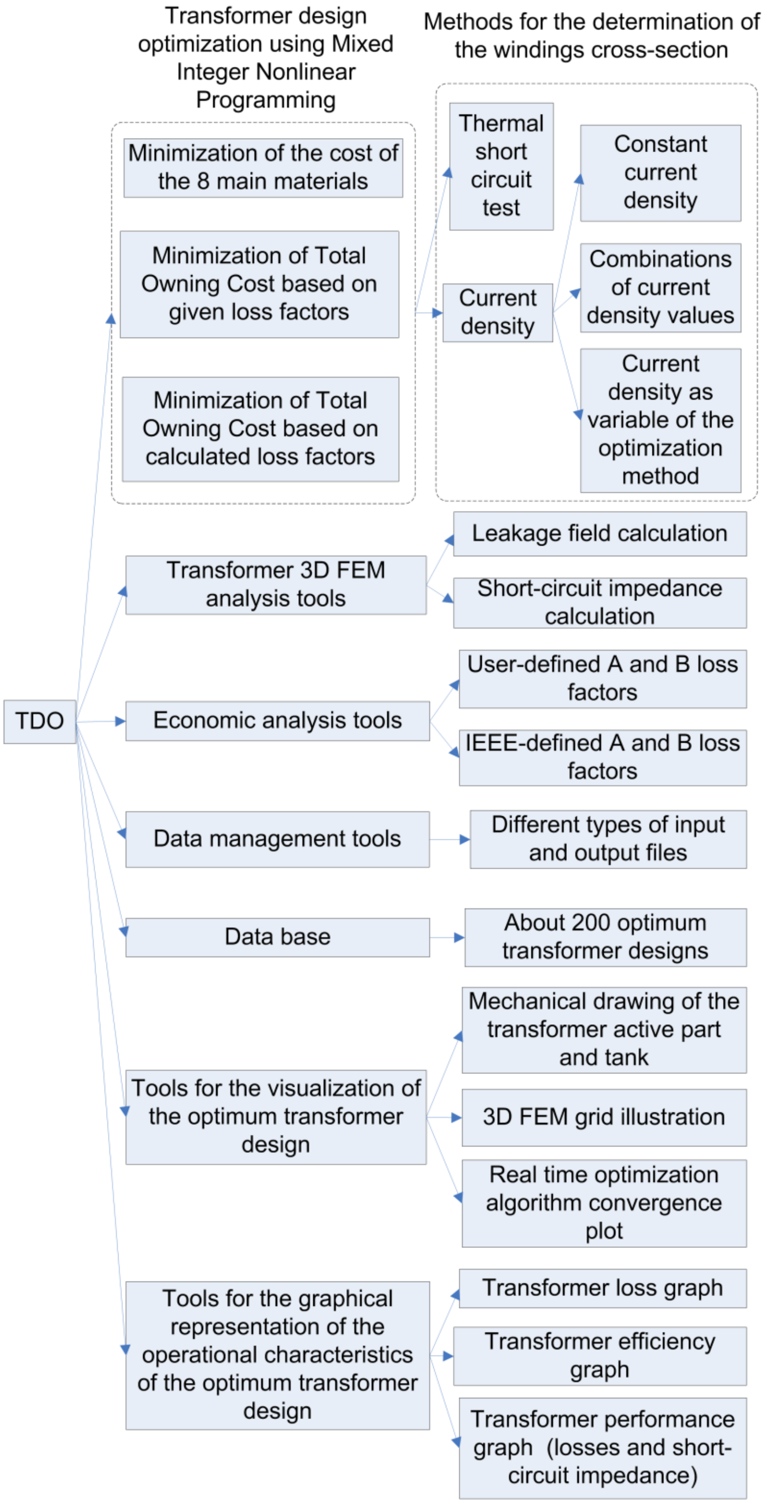


Fig. 1 Flowchart of the proposed software package.

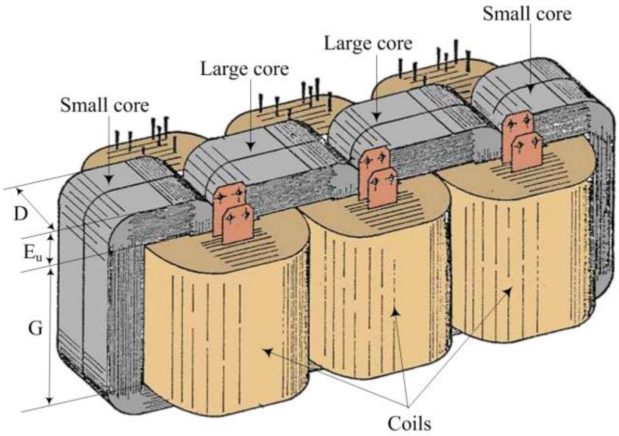
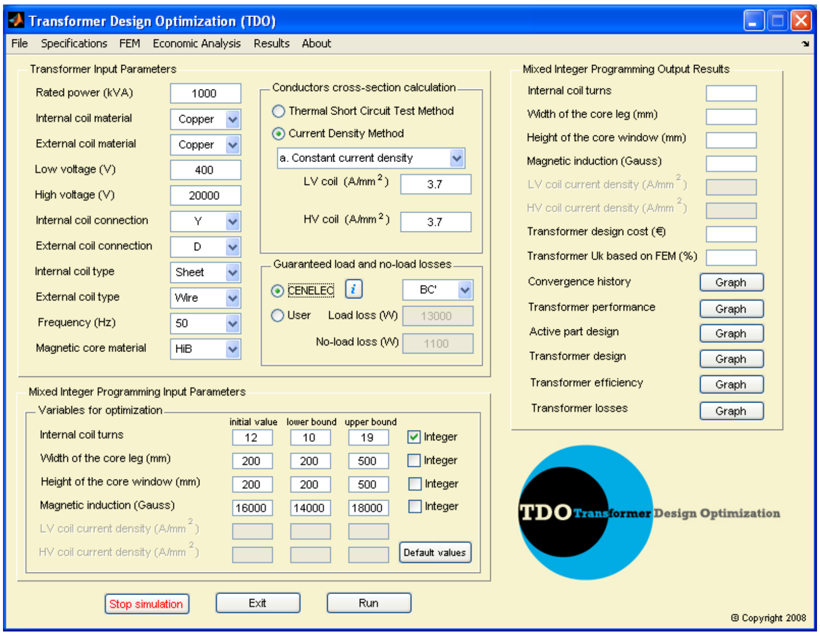


Fig. 2. Active part configuration of the three-phase wound core power transformer considered [13].



Transformer Input Parameters

Rated power (kVA)	1000
Internal coil material	Copper
External coil material	Copper
Low voltage (V)	400
High voltage (V)	20000
Internal coil connection	Y
External coil connection	D
Internal coil type	Sheet
External coil type	Wire
Frequency (Hz)	50
Magnetic core material	HiB

Conductors cross-section calculation

○ Thermal Short Circuit Test Method
● Current Density Method
a. Constant current density
LV coil (A/mm^2) 3.7
HV coil (A/mm^2) 3.7

Guaranteed load and no-load losses

● CENELEC BC'
○ User Load loss (W) 13000
No-load loss (W) 1100

Mixed Integer Programming Input Parameters

Variables for optimization

	initial value	lower bound	upper bound	
Internal coil turns	12	10	19	☑ Integer
Width of the core leg (mm)	200	200	500	☐ Integer
Height of the core window (mm)	200	200	500	☐ Integer
Magnetic induction (Gauss)	16000	14000	18000	☐ Integer
LV coil current density (A/mm^2)				
HV coil current density (A/mm^2)				Default values

Mixed Integer Programming Output Results

Internal coil turns	
Width of the core leg (mm)	
Height of the core window (mm)	
Magnetic induction (Gauss)	
LV coil current density (A/mm^2)	
HV coil current density (A/mm^2)	
Transformer design cost (€)	
Transformer Uk based on FEM (%)	
Convergence history	Graph
Transformer performance	Graph
Active part design	Graph
Transformer design	Graph
Transformer efficiency	Graph
Transformer losses	Graph

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Fig. 3. Graphical user interface of the proposed software package.

Methods for the determination of the windings cross-section

The windings cross-section is one of the most crucial design parameters, as explained in Section "Data Input". Its selection affects significantly the overall transformer design, since it is linked directly to the total winding area and indirectly to the core dimensions. The winding cross-section can be calculated by TDO software, with the use the thermal short-circuit test method. This method defines the minimum cross-section so that the windings can thermally withstand a short-circuit, however, it often results to over-estimated cross-sections and sub-optimal designs. For this purpose, an alternative way to define the windings cross-section through their current density is available (Fig. 1). The TDO user can select among three possibilities for the current density determination [11]: i) definition of a constant primary and secondary winding current density, ii) definition of an interval and step for the variation of the primary and secondary winding current density and calculation of all the respective combinations and iii) insertion of the primary and secondary winding current density to the design vector.

Constant current density. At the first approach, the transformer designer can define directly the value of the primary (HV) winding and secondary (LV) winding (in A/mm^2), denoted as WCD_{HV} and WCD_{LV}, respectively. The main drawback of this approach is that the transformer designer should be quite experienced in order to correctly set this value and direct the method to the optimal solution.

Combinations of current density values. At the second approach, an interval with a set of discrete c_{LV} and c_{HV} values for the primary (LV) and secondary (HV) winding, respectively, can be defined. In this case, the proposed method will calculate $c_{LV} \cdot c_{HV}$ optimum transformer designs, and finally will keep the best optimum transformer design among them. Although this approach is time-consuming, it assures a global optimum design.

Current density as variable of the optimization method. At the third approach, the designer can increase the vector of the four design variables into six. In particular, the correct definition of the current density value is under the rules (supervision) of the MINLP optimization method. In this way, the transformer designer defines the initial, the upper and the lower value of the WCD_{HV} and WCD_{LV} and the proposed method finds an optimum transformer design, designating the values of the six variables of the design vector.

Transformer 3D FEM analysis tools

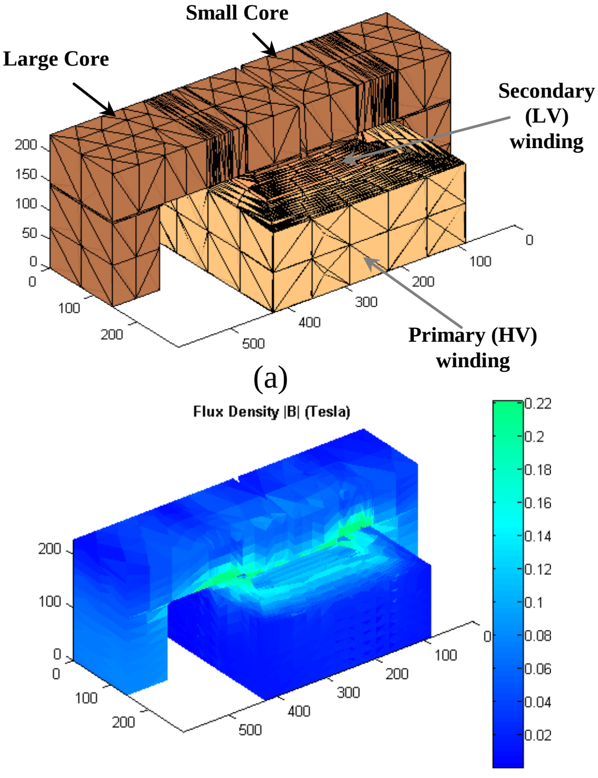
After the execution of the MINLP algorithm and the derivation of the optimum design, a set of numerical analysis tools incorporated to the software provide the user the ability to verify its performance characteristics. These tools involve the pre-processing, solution and post-processing of a 3D magnetostatics FEM model as well as the automatic generation of proper geometry input files to be used by other commercial finite element solvers for the derivation of other analyses (e.g. thermal analysis) of particular interest to the user.

The 3D FEM model incorporated in TDO software, developed in [19], is built automatically, using the geometrical data of the optimum solution and computes the transformer magnetostatic field under short-circuit test, according to its main electrical data. This analysis yields the transformer leakage field and short-circuit impedance, which are crucial operational parameters and provide a criterion for the verification that the optimum designs meets the imposed specifications.

In order to overcome problems related to the complexity of the 3D FEM mesh and the resulting increase in the execution time of the finite element calculations, the adopted model was designed with careful consideration to the detailed geometrical representation of the windings area, mainly affecting the leakage field calculation. Therefore, the model is able to produce accurate results with the use of small mesh densities, a characteristic that renders it quite suitable for incorporation to an integrated software package at a low computational cost. Moreover, it is entirely automated so that no user interaction is necessary during the definition of intricate details concerning the mesh generation (and other pre-processing tasks as the definition of materials and boundary conditions) as well as the finite element solver parameters definition.

Figure 4(a) illustrates the 3D FEM mesh of the transformer active part (confined to one fourth of the real geometry, due to the symmetries of the model, in order to reduce the mesh size and the total FEM execution time). Moreover, the possibility to visualize the leakage field results as density plot is provided to the user, as depicted in Fig. 4(b). Two mesh densities are available to the user, a sparse one (consisting of approximately 3000 nodes) and a mesh of intermediate density (consisting of approximately 10000 nodes), which corresponds to Fig. 4(b).

Apart the abovementioned 3D FEM analysis, additional tools for finite element analysis are incorporated in TDO software. These tools involve the automatic generation of proper two-dimensional (2D) geometry files that can be used as input in other commercial finite element solvers, as Finite Element Method Magnetics (FEMM) [20]. A number of alternative files with different discretization options in the core and windings area (i.e. geometries that can be used to produce more dense mesh either in the windings area, the cores area or both, according to which part needs to be modelled in greater detail) are automatically generated through the selection of the proper TDO menu. These 2D geometry files can be used to perform magnetostatic and thermal analysis, which can be of particular interest to the transformer designer.



(a)

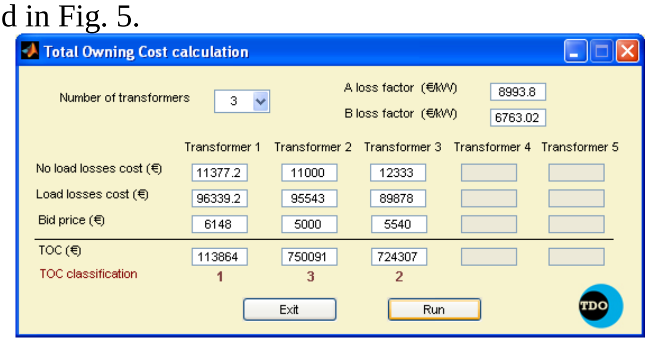
(b)

Fig. 4. Visualization of 3D FEM tools incorporated to TDO: (a) 3D FEM mesh and (b) leakage field distribution.

Transformer Economic Analysis Tools

The total owing cost technique provides transformer manufacturers and users with a robust tool for the economic evaluation of different transformer designs. It is incorporated in TDO software in order to enable its user to compare the optimum design with other designs (either sub-optimal designs or different designs defined by the user) and evaluate its energy efficiency.

For the proper TOC calculations the loss cost coefficients, namely factors A and B, used for the calculation of no-load and load loss cost, respectively, must be defined. The proposed software enables the definition of constant factors by the user or their calculation according to IEEE Standard C57.120 [18] (the same philosophy that was applied for the TOC calculation as objective function in Section “Data Input”). The considered transformer designs are then classified according to their TOC ranking as indicated in Fig. 5.



Total Owning Cost calculation

Number of transformers: 3

A loss factor (€/kW): 8993.8

B loss factor (€/kW): 6763.02

	Transformer 1	Transformer 2	Transformer 3	Transformer 4	Transformer 5
No load losses cost (€)	11377.2	11000	12333		
Load losses cost (€)	96339.2	95543	89878		
Bid price (€)	6148	5000	5540		
TOC (€)	113864	750091	724307		
TOC classification	1	3	2		

Fig. 5. Transformer economic evaluation based on TOC.

Visualization tools

The visualization tools integrated in TDO software provide a first approach for the evaluation of the quality of the optimum solution yielded by MINLP algorithm. They can be divided into three categories: i) the convergence plot of the optimization algorithm, ii) the mechanical drawings of the transformer active part and tank and iii) the graphs of the transformer performance characteristics.

Convergence of the optimization algorithm. The manufacturing cost (i.e. the cost of the transformer eight main materials) of the current solution (i.e. the design corresponding to a set of values for the design variables examined at the current iteration of the optimization algorithm) is plotted online during the optimization process, as illustrated in Fig. 6. This plot provides a measure of the difficulty in the location of the global optimum, which can be exploited by the designer as information for fine tuning of the MINLP algorithm parameters of future designs. Moreover, it can be used as a criterion to terminate the execution in cases where the number of iterations increases significantly or no convergence to the optimum seems to be achieved.

Geometrical characteristics of the optimum transformer design. The 2D mechanical drawing of the transformer active part, and both the transformer active part and tank (Fig. 7) are generated by the software, along with the respective axes for the visualization of their real dimensions. This drawing provides an overview of the main proportions of the transformer basic components (windings, core and tank), indicating if the optimum design is easy to implement in the production line, or corresponds to a specialized design that is hard to be manufactured (therefore an expert design engineer may exploit it to decide whether to keep this optimum solution or repeat the optimization process after modification of some of the input parameters). It can also be used as a guideline during the manufacturing process.

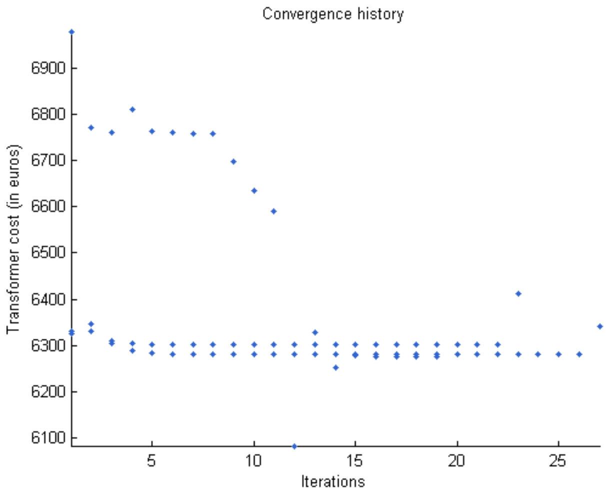


Fig. 6. Real time optimization algorithm convergence plot.

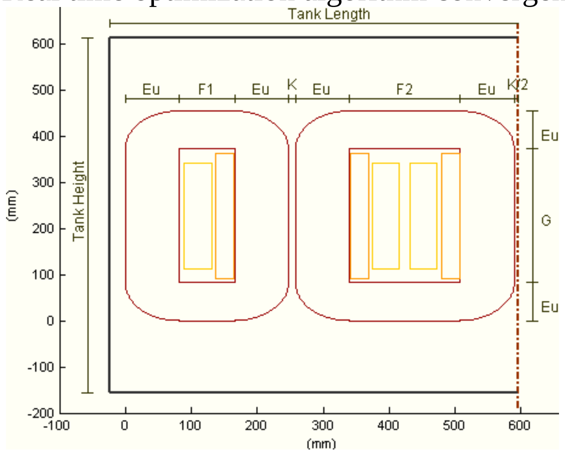


Fig. 7. The 2D cross-section of the optimum transformer active part and tank. The small and the large core are shown in dark red, while the primary and secondary winding are shown in yellow and orange, respectively.

Performance characteristics of the optimum transformer design. The performance of the optimum transformer design is evaluated through calculation of its efficiency and total losses for various transformer loading conditions, as depicted in Fig. 8. Moreover, the bar chart of Fig. 9 can be plotted, indicating the relative difference between the designed and guaranteed values of losses and short-circuit impedance.

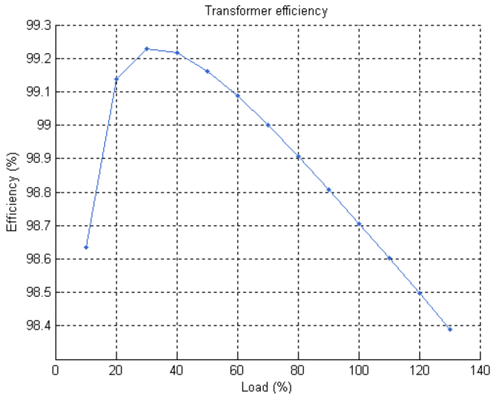
Results and Discussion

The proposed method has been applied in a wide spectrum of actual transformers, of different voltage ratings and loss categories. In particular, 188 optimum transformer designs were created and compared with the heuristic optimization methodology [15]. It should be noted that the aforementioned experiments were carried out using constant WCD_{HV} and WCD_{LV} values (first approach for the current density determination, described in Section “Methods for the determination of the windings cross-section”) because the current heuristic technique [15] could not support the other two approaches.

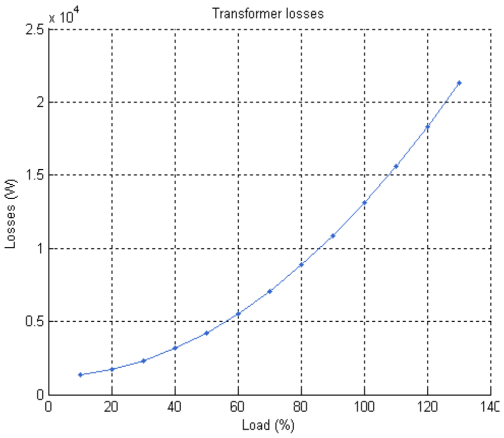
For the proper comparison of the heuristic and the proposed MINLP methodology, the optimum designs were compared on a common basis of input data, according to the following list:

- Guaranteed no-load and load losses and respective tolerances between the designed and guaranteed values;
- Cost of the eight main materials;
- Primary and secondary winding material;
- Primary and secondary voltage;
- Primary and secondary winding connection;
- Type of primary and secondary winding conductor;
- Primary and secondary winding current density;
- Frequency;
- Magnetic material;
- Common interval for the variation of the design variables (the number of secondary winding turns, the magnetic induction magnitude (B), the width of core leg (D) and the core window height (G) (Fig. 2)).

For the above input data, 188 optimum designs were produced, using both methodologies. More specifically: 14 designs of 1600 kVA rating, 24 designs of 1000 kVA rating, 20 designs of 800 kVA rating, 48 designs of 630 kVA rating, 28 designs of 400 kVA rating, 16 designs of 250 kVA rating, 24 designs of 160 kVA rating and 14 designs of 100 kVA rating. Fig. 10 depicts the mean manufacturing cost (i.e. cost of the main eight materials) difference between the heuristic and the MINLP methodology. According to Fig. 10, TDO provides an average cost difference of 1.60 % for the sum of the considered 188 designs. This means that the use of TDO results to optimum designs that are on average 1.60% cheaper than the ones provided by the heuristic methodology. This cost difference is achieved without compromises to the quality of the optimum designs, as far as conformity to the design specifications and the rest of the performance parameters are concerned.



(a)



(b)

Fig. 8. Graph of the optimum transformer operational characteristics as a function of its % loading.
(a) Efficiency graph
(b) Total losses graph

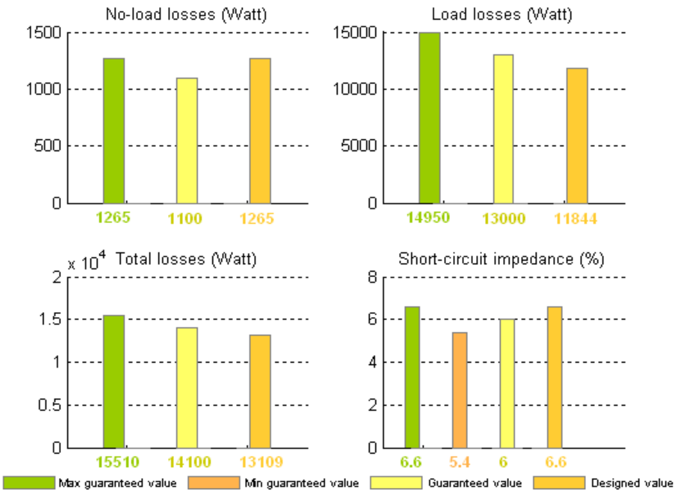


Fig. 9. Bar chart of the optimum transformer designed no-load losses, load losses, total losses and short-circuit impedance, with respect to the guaranteed values.

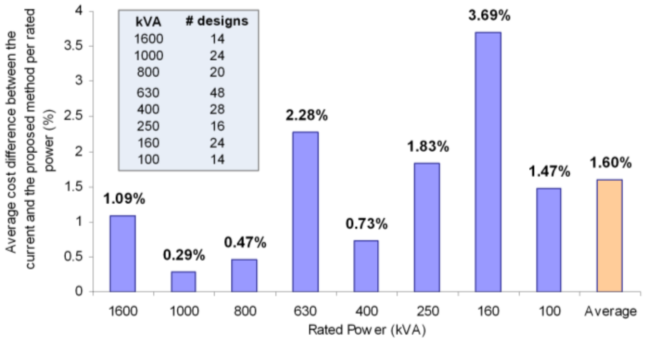


Fig. 10. Average cost difference between the heuristic optimization methodology and the MINLP methodology implemented by TDO.

Conclusion

In the present article, a complete software package for transformer design optimization was presented, consisting a collection of design optimization, visualization and verification tools, able to provide transformer designers all the proper interactive capabilities required for the enhancement of the automated design process of a manufacturing industry. The main advantages of the software are: i) minimization of the necessary input data and user interaction, enabling a deskilled design optimization process, accessible to engineers with little experience in transformer engineering, ii) adoption of a robust deterministic optimization methodology, in conjunction with novel techniques for the definition of crucial transformer technical characteristics, ensuring convergence to global optimum, iii) incorporation of numerical field analysis tools for the verification of the optimum designs, iv) integration of economic evaluation tools for the assessment of the energy efficiency of the optimum designs, v) tools for the visualization and evaluation of the performance characteristics of the optimum solutions, vi) creation of a large database of optimum designs of various ratings that can be used as a reference by the user and vii) convenience in the management of input and output data, through proper data management tools. Moreover, the efficiency of the software is illustrated by its application to a wide spectrum of actual transformers, of different power ratings and losses, resulting to optimum designs with an average cost saving of 1.60% in comparison with the existing heuristic method used by a transformer manufacturer.

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